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Forecasting comparison of long term component dynamic models for realized covariance matrices

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November 2014

Abstract

Novel model specifications that include a time-varying long run component in the dynamics of realized covariance matrices are proposed. The adopted modeling framework allows the secular component to enter the model structure either in an additive fashion or as a multiplicative factor, and to be specified parametrically, using a MIDAS filter, or non-parametrically. Estimation is performed by maximizing a Wishart quasi-likelihood function. The one-step ahead forecasting performance of the models is assessed by means of three approaches: the Model Confidence Set, (global) minimum variance portfolios and Value-at-Risk. The results provide evidence in favour of the hypothesis that the proposed models outperform benchmarks incorporating a constant long run component, both in and out-of-sample.

Keywords: Realized covariance, component dynamic models, MIDAS, minimum variance portfolio, Model Confidence Set, Value-at-Risk.

JEL Classification: C13, C32, C58.

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1 Introduction

Volatility models that include a time-varying long run component have been increasingly used over the last decade. Empirical evidence has indeed suggested that the level of variances and correlations is changing over time as a function of economic conditions. This can have important forecasting implications, as models reverting to a constant level in the long run will produce the same long term forecast at any point in time, thus completely ignoring the changing economic conditions. Without controlling for the different sources affecting volatility, they will end up producing spurious forecasts.

Since Engle & Lee (1999) introduced a GARCH model with additive long run and short run dynamic components, several others have proposed related component models for volatility. Engle & Rangel (2008) specify a semi-parametric model that allows for time-variation in the unconditional level of stock market volatility, while Engle et al. (2008) use a parametric specification involving the Mixed Data Sampling (MIDAS) scheme to extract a slowly moving secular component around which daily volatility moves. Specifically, the short run component is a GARCH specification based on daily squared returns, while the long run component is driven by realized volatilities computed over a monthly, quarterly or biannual basis. The idea has been extended to dynamic correlations in the DCC-MIDAS of Colacito et al. (2011) and in the multiplicative-DCC of Bauwens et al. (2013).

More recently, researchers have paid attention to the specification of models directly fitted to time series of realized covariances. As advocated by Andersen et al. (2003), high-frequency based models produce significant improvements in predictive performance relative to models that rely on daily data alone. Pioneering works in this respect can be found in Gouriéroux et al. (2009), Jin & Maheu (2013), and Chiriac & Voev (2011), among others. Our approach is closer to Golosnoy et al. (2012), who proposed a model where the realized covariance matrix of asset returns is assumed to follow a conditional Wishart distribution with a time-varying conditional expectation that is inspired by the BEKK model of the multivariate GARCH literature. This conditionally autoregressive Wishart (CAW) model aims to capture the daily dynamics of the realized covariances around a constant level matrix. Golosnoy et al. (2012) also propose an extension of their basic model specifically designed to capture long run fluctuations in the levels of covariances, by replacing the constant level matrix by a MIDAS filter of past realized covariance matrices. They apply their models to daily realized covariance matrices for the returns of five stocks and show that the MIDAS-CAW specification dominates the baseline CAW model both in-sample and out-of-sample. That application involves estimating more than one hundred parameters due to the use of a full BEKK specification and thus it is difficult to apply this model in a higher dimensional setting.

Inspired by this approach, we propose a wider set of models for realized covariance matrices which account both for time variation in the long run levels of (co)volatilities and for their short-run dynamics around these levels, and are applicable in higher dimensions.

Similarly to Golosnoy et al. (2012), we decompose the conditional expectation of the realized covariance matrix into a slowly evolving process and a mean-reverting short run one. We extend the existing framework discussed so far mainly into two directions. First, our modeling setting allows the long-run component to enter the model structure either in additive fashion, as a time-varying intercept in the volatility and correlation (or covariance) models, or as a multiplicative factor. This allows us to specify it parametrically, using a MIDAS-type filter, or even non-parametrically, by means of a matrix-variate smoother. Second, we exploit the advantages of a parsimonious modeling of the short run dynamics by employing different specifications inspired by the multivariate GARCH literature, namely scalar BEKK, DCC and DECO-type models.

The estimation of the models is performed by maximizing in one step the likelihood function based on the conditional Wishart assumption. As shown in Bauwens et al. (2012) and Noureldin et al. (2012), the function has a quasi-likelihood interpretation and the estimator is consistent even if the assumed underlying distribution is not Wishart.

The remainder of the paper is organized as follows. In Section 2 we introduce the set of time-varying long term component models and we explain in detail the different component structures employed. The procedure for quasi-maximum likelihood (QML) estimation of the proposed models is covered in Section 3. Section 4 presents the results of an empirical application to a set of ten U.S. stocks. Specifically, we first compare the in-sample goodness of fit of the different models in terms of standard information criteria, then we evaluate their forecasting performance by means of three approaches: the Model Confidence Set, (global) minimum variance portfolios and Value-at-Risk. Overall, our results illustrate the potential benefits deriving from using models that include a time-varying long run component against models incorporating a constant long run component. Section 5 concludes the paper with some final remarks.

2 Component models for realized covariance matrices

The purpose of this section is to formally present the new multivariate time-varying long term component dynamic models for realized volatilities and correlations of asset returns. Within each subsection, models incorporating a constant long run component are derived as special cases. They will be used as benchmarks in the empirical analysis. As a first step, in the next subsection, we define the common modeling framework and provide some preliminary notations.

2.1 General framework

Let C_t be a positive definite and symmetric (PDS) daily realized covariance matrix of order n. We assume that conditionally on past information I_{t-1} consisting of C_τ for $\tau \leq t-1$, C_t follows a n-dimensional central Wishart distribution:

$$C_t|I_{t-1} \sim W_n(\nu, S_t/\nu), \quad \forall t = 1, \ldots, T \tag{1}$$

where ν ($> n-1$) is the degrees of freedom parameter and S_t/ν is a PDS scale matrix of order n. From the properties of the Wishart distribution – see e.g. Anderson (1984) – it follows that the conditional mean is

$$E(C_t|I_{t-1}) = S_t, \tag{2}$$

so that the matrix S_t is referred to as the conditional covariance matrix of r_t in the sequel, and any of its off-diagonal element as a conditional covariance (or variance for a diagonal element).

Equations (1)-(2) define a generic conditional autoregressive Wishart (CAW) model as proposed by Golosnoy et al. (2012). A CAW model specifies the dynamic dependence of the $\{C_t\}$ process through a dynamic equation for the scale matrix S_t of the Wishart distribution, so as to capture the serial dependences in the realized variances and covariances. In addition, S_t can be further designed to directly capture the long run fluctuations in the levels around which realized variances and covariances or correlations fluctuate from day to day. To this extent, the realized covariance dynamics are split in two components, a secular smoothly varying component and a short lived one. In the next subsections, we define several ways to specify both components.

2.2 Long term additive component models for correlations

The structure of the first set of models is inspired by the DCC-MIDAS model of Colacito et al. (2011). In that model, conditional variances and correlations are modelled separately. We retain the original model structure but we adapt it to the realized covariance framework. More specifically, the scale matrix S_t in Eq.(2) is decomposed in terms of standard deviations and correlations:

$$S_t = D_t R_t D_t, \tag{3}$$

where the i-th entry of the diagonal matrix $D_t = \{\text{diag}(S_t)\}^{1/2}$ is given by the conditional standard deviation $\sqrt{S_{ii,t}}$ of asset i, and R_t is the corresponding conditional correlation matrix of r_t. Notice that the terms 'conditional' are used for convenience and do not imply that $\sqrt{S_{ii,t}}$ is the conditional mean of $\sqrt{C_{ii,t}}$, and that R_t is the conditional mean of the correlation matrix obtained from C_t. This structure allows us to model separately conditional variances and correlations, in the spirit of Engle (2002) and Tse & Tsui (2002). Moreover, by ruling out spillover effects between conditional variances, each univariate volatility model can be specified and estimated independently of the others.

Baseline models Short and long term dynamic components are accounted for, but separately treated. Namely, for every t, each conditional volatility is expressed as the product of two components:

$$E(C_{ii,t}|I_{t-1}) = \bar{S}_{ii,t}S_{ii,t},$$

with $\bar{S}_{ii,t}$ and $S_{ii,t}$ denoting the secular and the mean-reverting short run component, respectively. For each asset $i = 1, ..., n$, the short run component is specified as a mean reverting GARCH(1,1)-type process:

$$S_{ii,t} = (1 - \gamma_i - \delta_i) + \gamma_i \frac{C_{ii,t-1}}{\bar{S}_{ii,t-1}} + \delta_i S_{ii,t-1}, \tag{4}$$

where mean-reversion to unity is imposed for identification and stationarity is assured by imposing the usual restriction $\gamma_i + \delta_i < 1$.

The secular component is specified using a MIDAS filter assumed to be a weighted sum of a judiciously chosen number L of lagged realized variances:

$$\bar{S}_{ii,t} = \bar{m}_i + \theta_i \sum_{l=1}^{L} \phi_l(\omega_s) C_{ii,t-l}, \qquad l = 1, ..., L, \tag{5}$$

where $\bar{m}_i$ and θ_i are restricted to be positive scalars. Differently from Colacito et al. (2011), who specify $\bar{S}_{ii,t}$ as a locally constant long run component, we define it based on rolling samples that change from day to day. The weight function $\phi_l(\omega_s)$, normalized so that the sum of the weights is equal to one, is specified according to the Beta function, defined as

$$\phi_l(\omega_s) = \frac{\left(\frac{l}{L}\right)^{\omega_{1s}-1}\left(1 - \frac{l}{L}\right)^{\omega_{2s}-1}}{\sum_{j=1}^{L}\left(\frac{j}{L}\right)^{\omega_{1s}-1}\left(1 - \frac{j}{L}\right)^{\omega_{2s}-1}}, \tag{6}$$

where for brevity, $\phi(\omega_s) = \phi(\omega_{1s}, \omega_{2s})$. We use the subscript s to differentiate ω_s from the similar scheme that is introduced below for correlations. In order to ensure a decaying pattern of the weights, it suffices to impose $\omega_{1s} = 1$ and estimate ω_{2s} under the restriction $\omega_{2s} > 1$; after this, the decaying speed of the weights over time is determined by the data. It should be obvious that the number of lags L can be chosen high enough so as to obtain a sufficiently smooth path of the long run component. In order to avoid truncating at too low a lag, which could hide important long run dependences, we set L equal to 264 so as to cover a period of one year.

The component structure for the conditional correlation matrix is inspired by the same ideas. The main element is to replace the constant intercept matrix in the DCC specification by a time-varying component expressed as a weighted average of past realized correlations. This amounts to specify R_t as

$$\begin{aligned} R_t^{DCC} &= (1-\alpha-\beta)\bar{P}_t + \alpha P_{t-1} + \beta R_{t-1}^{DCC}, &(7)\\ \bar{P}_t &= \sum_{l=1}^{L} \phi_l(\omega_r) P_{t-l}, &(8) \end{aligned}$$

where $P_t = \text{diag}\{C_t\}^{-1/2} C_t \text{diag}\{C_t\}^{-1/2}$ and $\phi_l(\omega_r)$ is again a Beta weight function that applies the same weighting scheme to all the elements of the matrix P_t.

Instead of using the DCC-type specification, the dynamic equicorrelation (DECO) type of model of Engle & Kelly (2012) can also be used. It is written as

$$\begin{aligned} R_t^{DECO} &= (1-\rho_t) I_n + \rho_t J_n, &(9)\\ \rho_t &= \frac{1}{n(n-1)} \left(\iota' R_t^{DCC} \iota\right), \end{aligned}$$

where ρ_t is the equicorrelation parameter at time t and ι denotes a vector of ones.

Equations (4) to (8) denote the *Additive Midas Realized DCC (AMReDCC)* model, while replacing Eq. (8) with Eq. (9) gives the *AMReDECO* model. Note that by imposing $\theta_i = 0$ in every univariate volatility equation and assuming a constant long run level for correlations (i.e. $\bar{P}_t = \bar{P}\ \forall t$), one obtains the RDCC model of Bauwens et al. (2012) as a particular case. The RDCC can thus be considered as a benchmark for this class of models, even though it is not formally nested in the AMReDCC since the latter does not include a constant intercept matrix in Eq.(7).

Variations of the baseline models Two variations of the baseline models are proposed below. To start with the conditional volatility dynamics, we propose an alternative specification for the MIDAS polynomial in Eq.(5), in the spirit of Golosnoy et al. (2012). Specifically, we aggregate the univariate series of realized variances over a horizon of m trading days and consider K lagged observations, such that Eq.(5) is rewritten as follows:

$$\begin{aligned} \bar{S}_{ii,t} &= \bar{m}_i + \theta_i \sum_{k=1}^{K} \phi_k(\omega_s) C_{ii,t,k}^{(m)} &(10)\\ C_{ii,t,k}^{(m)} &= \sum_{\tau=t-mk}^{t-m(k-1)-1} C_{ii,\tau}, \qquad k=1,...,K. \end{aligned}$$

Clearly, for Equations (5) and (10) to be comparable, one must impose K sufficiently smaller than L. Namely, by considering a span length of the aggregation period m equal to 22 days and a number of lags K equal to 12, the series of daily realized variances are aggregated over a period of one year, thus covering the same time span as in Eq.(5).

A similar variation is applied to the MIDAS component driving the long run correlation dynamics. The matrix $\bar{P}_t$ in Eq.(8) is replaced by a weighted sum of past realized correlation matrices aggregated over the past year, i.e.

$$\begin{aligned} \bar{P}_t &= \sum_{k=1}^{K} \phi_k(\omega_r) P_{t,k}^{(m)} \\ P_{t,k}^{(m)} &= \sum_{\tau=t-mk}^{t-m(k-1)-1} P_\tau. \end{aligned} \tag{11}$$

This definition of the time-varying intercept matrix $\bar{P}_t$ does not yield a correlation matrix and consequently, R_t^{DCC} needs to be rescaled into a conditional correlation matrix. Using Equations (7)-(8), one just needs to compute

$$R_t^{DCC*} = diag\{R_t^{DCC}\}^{-1/2} R_t^{DCC} diag\{R_t^{DCC}\}^{-1/2}. \tag{12}$$

This new model is defined as *Additive Midas Aggregated Realized DCC (AMAReDCC)*, or *AMAReDECO* if the DECO version is used.

The second variation proposed for the specification of the conditional correlation matrix stems from the fact that the construction of R_t^{DCC} in Eq.(7) builds upon daily realized covariances filtered from realized standard deviations rather than conditional standard deviations. By purging the realized covariance matrices from the conditional standard deviations, one can eliminate potential heteroskedasticity left in C_t and obtain a potentially cleaner estimator. Namely, C_t can be transformed to

$$C_t^p = D_t^{-1} C_t D_t^{-1}, \qquad t = 1, ..., T, \tag{13}$$

and then used to compute the time-varying intercept matrix $\bar{P}_t$ in Eq. (7). This resembles the approach used in the two-step estimation of the DCC model of Engle (2002), where the outer product of the first-stage (estimated) standardized residuals ξ_t is used to compute the second-stage conditional correlation matrix $R_t = E_{t-1}[\xi_t \xi_t']$.

By combining the different equations proposed for the conditional variances and correlations with the two structures of the MIDAS filter, we obtain eight additive models that are summarized in Table 1.

Table 1: List of models with additive MIDAS component for correlations

Models	Aggregated	Purged	Equations
1 - AMReDCC			(4)-(5)-(7)-(8)
2 - AMReDECO			(4)-(5)-(7)-(9)
3 - AMAReDCC	✓		(4)-(10)-(7)-(11)-(12)
4 - AMAReDECO	✓		(4)-(10)-(7)-(11)-(9)
5 - AMReDCC-P		✓	(4)-(5)-(13)-(7)-(8)
6 - AMReDECO-P		✓	(4)-(5)-(13)-(7)-(9)
7 - AMAReDCC-P	✓	✓	(4)-(10)–(13)-(7)-(11)-(12)
8 - AMAReDECO-P	✓	✓	(4)-(10)–(13)-(7)-(11)-(12)-(9)

Notes – The suffix 'P' denotes the models built on the purged estimator in Eq. (13). The number of parameters is equal to $5n + 3$ in each model, where n is the number of assets.

2.3 Long term multiplicative component models

The component models that have been suggested in the most recent literature are mainly based on a multiplicative structure. Examples are the Spline-GARCH model of Engle & Rangel (2008), the multiplicative BEKK model of Hafner & Linton (2010), the multiplicative-DCC model of Bauwens et al. (2013) and the MIDAS-CAW model of Golosnoy et al. (2012). Our approach builds on these contributions. The proposed component models feature a multiplicative decomposition of the conditional covariance matrix S_t into a secular component $M_t = L_t L_t'$ and a short-lived component $S_t^\star$, as follows:

$$S_t \;=\; L_t S_t^\star L_t'. \tag{14}$$

The matrix square root L_t can be obtained by a Cholesky factorization of the long term component matrix M_t.

The main difference with the class of additive models presented in Section 2.2 is that the multiplicative structure allows the matrix M_t to be modelled either via a parametric function of past realized covariances or via a nonparametric function of time. Another difference is that a unique model is specified for the long term components of the realized variances and covariances altogether, instead of different models for realized variances and correlations.

Baseline models In the baseline parametric multiplicative models, the secular component is specified as a multivariate generalization of the MIDAS filter used in Eq. (5). This can be written as

$$M_t = \bar{\Lambda} + \theta \sum_{l=1}^{L} \phi_l(\omega_r) C_{t-l}, \tag{15}$$

where the first term $\bar{\Lambda}$ is a n×n PDS matrix of constant parameters, θ is a positive scalar parameter and $\omega_r = (\omega_{1r}, \omega_{2r})$ is the vector of weights of the Beta function obtained under the restrictions $\omega_{1r} = 1$ and $\omega_{2r} > 1$. This long term component has $n(n+1)/2+2$ parameters. It is worth to emphasize that in sufficiently low dimensions this does not represent an issue, but the proliferation of parameters in large dimensions will eventually render impossible numerical maximization of the log-likelihood function. Note that by imposing $\theta = 0$ in the MIDAS specification of Eq. (15), the secular component becomes time invariant, being limited to the constant intercept matrix $\bar{\Lambda}$. This constant long term structure is considered as a benchmark in the empirical application.

A nonparametric formulation consists in letting the long run component matrix M_t be a smooth unknown function of the rescaled time index, i.e. $M_t = M(t/T)$. This assumption implies that the unconditional covariance varies over time, since using $E\left[E(C_t|I_{t-1})\right] = E(S_t)$ and imposing $E(S_t^\star) = I_n$ for identification (see below), it follows that $E(L_t S_t^\star L_t') = L_t L_t' = M_t$. One could also assume the matrix M_t to be a smooth function of an observable variable x_t (for example a market volatility index), such that $M_t = M_t(x_{t-1})$.

The multiplicative structure in Eq. (14) allows for different specifications to be used for modeling the dynamics of $S_t^\star$. In this context, BEKK, DCC and DECO parameterizations apply to the short run (co)volatility components independently of the parametric or nonparametric structure assumed for the long run component. When the last two are used, S_t^* is written as

$$S_t^\star \quad = \quad D_t^\star R_t^\star D_t^\star \tag{16}$$

where $D_t^\star = \{\text{diag}(S_t^\star)\}^{1/2}$ is the diagonal matrix of short term conditional standard deviations and $R_t^\star$ is the corresponding short term conditional correlation matrix.

DCC and DECO allow for a separate treatment of conditional volatilities and correlations, and their scalar specifications correspond to the following equations:

$$\begin{aligned} S_{ii,t}^\star &= (1-\gamma_i-\delta_i) + \gamma_i C_{ii,t-1}^\star + \delta_i S_{ii,t-1}^\star, & (17)\\ R_t^{DCC\star} &= (1-\alpha-\beta)I_n + \alpha P_{t-1}^\star + \beta R_{t-1}^{DCC\star}, & (18)\\ R_t^{DECO\star} &= (1-\rho_t)I_n + \rho_t J_n, & (19)\\ \rho_t &= \frac{1}{n(n-1)}\left(\iota' R_t^{DCC\star}\iota\right), & (20) \end{aligned}$$

where

$$P_t^\star = \{\text{diag}(C_t^\star)\}^{-1/2} C_t^\star \{\text{diag}(C_t^\star)\}^{-1/2}$$

and

$$C_t^\star = L_t^{-1} C_t (L_t')^{-1}.$$

The matrix $C_t^\star$ is the realized covariance matrix purged of its long term component, and the matrix $P_t^\star$ is the corresponding short term realized correlation matrix.

Finally, the scalar BEKK specification can be written as

$$S_t^\star = (1-\alpha-\beta)I_n + \alpha C_{t-1}^{*} + \beta S_{t-1}^\star. \tag{21}$$

Note that, like in the additive framework, mean reversion to unity in Eq.(17) and $E(S_t^\star) = I_n$ in Equations (18) and (21) are imposed as identifying restrictions.

Variations of the baseline models The single variation applied to this set of models consists in the aggregation of the series of realized covariance matrices in the MIDAS filter. For completeness, this amounts to specify the long term component M_t as

$$\begin{aligned} M_t &= \bar{\Lambda} + \theta \sum_{k=1}^{K} \phi_k(\omega_r) C_{t,k}^{(m)}, \\ C_{t,k}^{(m)} &= \sum_{\tau=t-mk}^{t-m(k-1)-1} C_\tau \end{aligned} \tag{22}$$

where the constants m and K are set in the way discussed after Eq.(10).

In total, we have defined nine models in the multiplicative family. The baseline models are either labeled *Multiplicative Midas Realized DCC (MMReDCC)* or *NonParametric Realized DCC (NPReDCC)*, with variations according to the assumed short run model type (DECO or BEKK) and MIDAS filter definition. A summary of these models is given in Table 2.

Table 2: List of time-varying multiplicative component models

	Models	Num. pars	Aggregated	Parametric	Nonparametric	Equations
9 -	NPReDCC	2n+3			✓	(24)-(17)-(18)
10 -	NPReDECO	2n+3			✓	(24)-(17)-(20)
11 -	NPReBEKK	3			✓	(24)-(21)
12 -	MMReDCC	[n(n+1)/2]+2n+3		✓		(15)-(17)-(18)
13 -	MMReDECO	[n(n+1)/2]+2n+3		✓		(15)-(17)-(20)
14 -	MMReBEKK	[n(n+1)/2]+3		✓		(15)-(21)
15 -	MMAReDCC	[n(n+1)/2]+2n+3	✓	✓		(22)-(17)-(18)
16 -	MMAReDECO	[n(n+1)/2]+2n+3	✓	✓		(22)-(17)-(20)
17 -	MMAReBEKK	[n(n+1)/2]+3	✓	✓		(22)-(21)

Notes – The first three models include a nonparametric long run component, while the others include a parametric MIDAS filter. The benchmarks used for this class are obtained as special cases of the parametric models and are denominated Constant Realized DCC or DECO or BEKK models (CReDCC, CReDECO and CReBEKK for brevity).

3 Estimation

The parametric models can be estimated by the method of maximum likelihood (ML) in one step. In general, given the Wishart assumption made on C_t, the log-likelihood function for

T observations, $\ell_T(\Phi)$, where Φ is the finite-dimensional vector of model parameters to be estimated, is expressed as follows:

$$\ell_T(\Phi) = \bar{c}T - \frac{\nu}{2}\sum_{t=1}^{T}\left[\log(|S_t|) + \mathrm{tr}\big(S_t^{-1}C_t\big)\right], \tag{23}$$

where $\bar{c}$ depends only on ν, n, and C_t. Hence, the log-likelihood function depends on Φ (through the matrix S_t) only via the last two terms. This formula is general, in the sense that it does not depend on the particular specification assumed for the long term and the short term components. In particular it also holds for the constant long run component models.

The last two terms on the right hand side of Eq. (23) are linear in the parameter ν. Hence, the first order conditions for the estimation of the parameter vector Φ do not depend on ν, implying that Φ can be estimated independently of the value of ν. In addition, it is known that the estimator based on the maximization of the Wishart log-likelihood function has a QML interpretation, because if the conditional expectation of C_t is correctly specified, the score of the log-likelihood function in Eq. (23), evaluated at the true value of the parameters, is a martingale difference sequence (MDS). Thus, under appropriate regularity conditions, the ML estimator is consistent even if the underlying distribution of C_t is not Wishart. We refer to Bauwens et al. (2012) and Noureldin et al. (2012) for technical details regarding the MDS property of the score and QML asymptotic results in this context.

The motivation for applying estimation in one step is twofold. First, the one-step maximization of the log-likelihood function simplifies inference, since one can easily compute (robust) standard errors and model selection criteria, such as the Akaike (AIC) and the Bayesian information criterion (BIC). This makes the in-sample comparison of different models easy. Second, estimation strategies based on sequential maximization of different log-likelihood components typically provide inefficient estimators whose asymptotic distributions can be difficult to compute.

In large-dimensional applications, the one-step estimation can be computationally challenging, and multi-step estimation procedures could eventually be used to obtain estimates of model parameters. In these cases, in addition, targeting strategies could be applied to pre-estimate some of the parameters, thus further reducing the computational burden. A detailed exploration of these estimation techniques goes beyond the scope of the present paper and is left as an open issue for future research.

In the case of the multiplicative models that have a nonparametric long term component, a two-step approach must be used: first, a kernel smoother is used to estimate the long run component M_t, and then, conditional on the estimate of the M_t matrices, the parameters driving the short term dynamics are estimated by ML. To estimate the matrices M_t non-parametrically as a function of rescaled time, we make use of the Nadaraya-Watson kernel estimator, which is consistent under general conditions; see for example Härdle (2004). It takes the following form:

$$M_t(\tau) = \frac{\sum_{t=1}^{T} K_h\left(\frac{t}{T} - \tau\right) C_t}{\sum_{t=1}^{T} K_h\left(\frac{t}{T} - \tau\right)}, \tag{24}$$

where $\tau \in [0, 1]$, $K_h(.) = K(./h)/h$, $K(.)$ is the Gaussian kernel function and h the bandwidth parameter determining the amplitude of the movements captured by the long term component. The bandwidth selection is performed by the least squares cross-validation method. To implement this, we use six-month rolling covariance matrices as the reference for computing the squared differences and adopt the penalizing function approach involving the bandwidth selector of Rice (1984). This estimator uses the same bandwidth for all the elements of the realized covariance matrices, even if this could be relaxed by smoothing separately variances and correlations with a different bandwidth for each diagonal element and a single one for the whole correlation matrix. Since we use the model for forecasting purposes, we use a one-sided version of the kernel estimator which is coherent with the idea that the information set available at time t consists only of data up to time $t - 1$.

4 Empirical study

This section presents the results of an empirical application to a time series of realized covariance matrices for U.S. stocks. First, we present the dataset being used and the forecasting framework adopted for constructing one-step ahead covariance forecasts. Then we provide full-sample estimation results and compare the estimated models in terms of information criteria. Finally, we perform an out-of-sample forecasting comparison of the proposed models vis-a-vis the corresponding benchmark models characterized by a constant long run component.

4.1 Data and forecasting scheme

The empirical analysis is based on a series of daily realized covariance matrices of ten stocks included in the Dow Jones Industrial Average (DJIA) index. The dataset has been previously used by Noureldin et al. (2012) and can be downloaded online from the *Oxford Man Institute Realized Library*. The data have been cleaned according to the procedure of Barndorff-Nielsen et al. (2009) and the open-to-close realized covariance estimator has been constructed on five-minute intraday returns aggregated with subsampling. We refer to the cited paper for a detailed explanation on the features of the dataset and the construction of the realized estimator.

The included stocks are Bank of America (BAC), JP Morgan (JPM), International Business Machines (IBM) , Microsoft (MSFT), Exxon Mobil XOM), Alcoa (AA), American Express (AXP), Du Pont (DD), General Electric (GE) and Coca Cola (KO). The dataset runs from February 1, 2001 to December 31, 2009, providing a total of 2240 observations. Descriptive statistics are provided in a Web Appendix. Figure 1 shows a representative example of time series plots of the realized variances of two stocks, and the corresponding realized covariance and correlation.

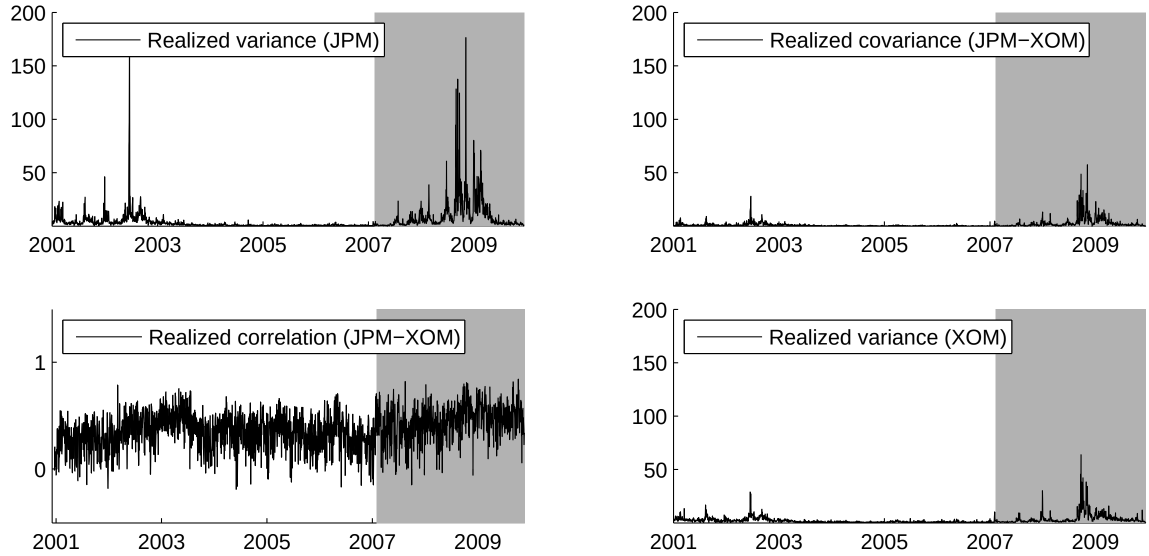

Figure 1: JPM and XOM realized volatility, realized covariance and realized correlation over the full-sample period 1/02/2001 – 31/12/2009. Out-of-sample period shaded in gray.

Our aim is to evaluate both the in- and out-of-sample performance of the dynamic component models against benchmark models that are based on constant long run components for volatilities and correlations. For the first evaluation, we estimate the proposed models using the complete sample and then we compare the fitted models by means of information criteria. For the second, one-day ahead daily covariance matrix forecasts are constructed. Specifically, the full dataset is divided into two different periods:

- Period I is the in-sample set for $t = 1, ..., 1528$. It corresponds to the relatively calm period from February 2001 to December 2006 and is reserved for the model initial estimation (before forecasting).

- Period II is the out-of-sample set comprising the remaining 712 observations used for the forecast evaluations. It is characterized by a higher unconditional volatility level stemming from the 2008-2009 crisis events included in the last part of the sample.

Forecasts are constructed using a fixed rolling window scheme that satisfies the assumptions

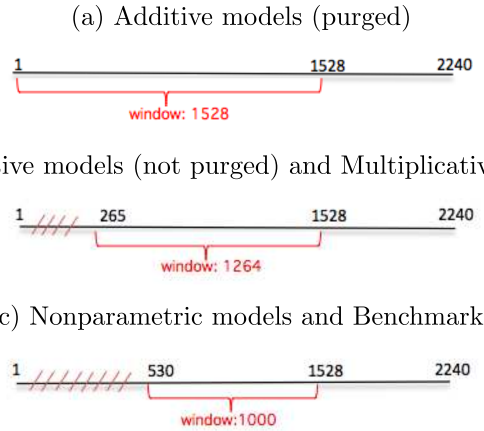

Figure 2: Implemented rolling window setting. The length of the in-sample rolling windows is set in order to get an effective number of 1000 observations and, depending on the specific model structure, three different window sizes are used. Specifically, additive models involving a purged estimator need 528 observations for initialization (Figure(a)) and hence they require a longer in-sample window of 1528 trading days. The remaining additive models and all the parametric multiplicative models (Figure(b)) need half the number of initial observations, thus the window is shifted forward of 264 days for a total of 1264 days. The models not incorporating a MIDAS component in the long-term structure (Figure(c)) do not loose any observation, hence the initial estimation starting point is shifted forward accordingly. T denotes the residual sample size after the trimming and is equal to 1712 trading days for all the models. The out-of-sample period starts at $t = 1529$ and covers the last 712 observations of the sample.

required by the MCS procedure and allows the comparison of nested models. In this respect, one clarification is necessary. The initialization of the MIDAS filter of the models discussed in Sections 2.2–2.3 requires some realized covariance matrices to be used as starting values; this amounts to reserve 529 initial observations for the additive models with purging and 264 for both the parametric multiplicative models and the remaining models in the additive group. No such loss of initial observations affects the nonparametric and the benchmark models, as their structure does not involve a MIDAS filtering scheme. Consequently, in order to obtain comparable in-sample statistics and parameter estimates, the estimation starting point and the in-sample window size need to be set differently according to the specific model structure. Figure 2 summarizes the implemented procedure. For each model, 712 forecasts are computed. We estimate the parameters of interest using the in-sample window and then we produce forecasts for the following 20 days, which approximately correspond to one month of trading. The estimation window is then shifted forward by 20 observations and the model is re-estimated. The new estimates are used to generate covariance forecasts for the subsequent 20 days and the whole procedure is repeated until all data until the end of December 2009 have been used. Irrespective of the initialization period, successive forecasts are effectively based on the most recent 1000 observations. Given the full sample of 2240 observations, the number of re-estimations of each model is equal to 36 but the last set of forecasts covers 12 days (period $t = 2228, ..., 2240$) instead of 20.

4.2 Full sample results

To evaluate the relative performance of our models in fitting the data we estimate them on the whole sample. Table B2 in the Web Appendix reports estimates of the parameters. The estimates of the MIDAS parameter θ of the multiplicative models are significant at the level of 5 percent at least, indicating the relevance of such a component. In most cases, when a time-varying long run component is included, the persistence of the correlation process (measured by the sum of α and β) is smaller than when the long run level is constant.

Table 3 reports the values of the maximized log-likelihoods for all the fitted models. For the fully parametric model specifications we also report the associated AIC and BIC values while the values of these criteria are not reported for models incorporating a nonparametric long run component. This choice is due to the fact that the penalty term appearing in the AIC and BIC formulas depends on the number of estimated parameters which cannot be explicitly determined since the long run component is estimated by means of a kernel smoother. Nevertheless, we provide in the table the values of the maximized log-likelihood functions of the three models having a nonparametric long run component.

The first interesting comparison is between the relative performance of the time-varying long run component models and their benchmarks. Except for some cases, the maximum values of the log-likelihoods are not comparable due to different numbers of parameters. In these cases we rely on the comparison of the estimated AIC and BIC values. Whether the criterion is AIC or BIC, each benchmark model is outperformed by the corresponding time-varying counterpart(s), a clear indication that assuming a constant long term structure for volatilities and correlations can be too restrictive when the market economic conditions change over time, as it is the case in the covered sample period: e.g. AIC of 35481 for MMReDCC, 35714 for CReDCC; BIC of 36049 for MMReBEKK, 36116 for CReBEKK; AIC of 35419 for AMReDCC-P, 35915 for RDCC.

Other comparisons are of interest. Within the additive model class, the comparison can be done by the log-likelihood values since all models have the same number of parameters. The models with a long term non aggregated structure strongly outperform their competitors, while the use of the purged estimator improves the fit only for the models with a long term non aggregated structure: the AMReDCC-P has the highest log-likelihood value but the improvement of 4 points over the AMReDCC is not spectacular.

In the multiplicative parametric class, there is no clear ranking between the models including different MIDAS filters (the difference in absolute value between one model and its corresponding aggregated version is only 15 points on average in terms of log-lik. values).

Table 3: Information criteria of the models

Models	Np	LogLik	AIC	BIC
1 - AMReDCC	53	-17661	35429	35717
2 - AMReDECO	53	-18093	36292	36581
3 - AMAReDCC	53	-17675	35456	35744
4 - AMAReDECO	53	-18110	36327	36595
5 - AMReDCC-P	53	*-17657*	*35419*	*35709*
6 - AMReDECO-P	53	-18078	36261	36551
7 - AMAReDCC-P	53	-17695	35785	35800
8 - AMAReDECO-P	53	-18112	36619	36633
9 - NPReDCC	–	**-17611**	–	–
10 - NPReDECO	–	-17726	–	–
11 - NPReBEKK	–	-17771	–	–
12 - MMReDCC	79	-17662	35481	35912
13 - MMReDECO	79	-17672	35501	35932
14 - MMReBEKK	59	-17805	35727	36049
15 - MMAReDCC	79	-17668	35494	35924
16 - MMAReDECO	79	-17696	35549	35980
17 - MMAReBEKK	59	-17790	35698	36019
18 - CReDCC	77	*-17780*	*35714*	36133
19 - CReDECO	77	-18184	36523	36942
20 - CReBEKK	57	-17846	35805	*36116*
21 - RDCC	22	-17881	35915	36335

Notes – 'Np' is the number of estimated parameters in each model. Maximized log-likelihood values are reported in the third column. Information criteria in columns 4 and 5 are computed as follows: $AIC = -2LogLik + 2Np$ and $BIC = -2LogLik + Np\log(T)$. The values in bold correspond to the globally best performing model while values in italics denote the best model in each category.

In a global ranking of the fully parametric models, three additive models (AMReDCC-P, AMReDCC, AMAReDCC) are ranked as the best fitting parametric models by AIC and BIC .

Concerning the specification of the short run component, within each class, DECO and BEKK models are always dominated by the corresponding DCC versions.

4.3 Forecasting results

The forecasting ability of the set of proposed models is evaluated over a series of 712 out-of-sample predictions. Figure 3 shows representative plots of the predicted variances, covariance and correlation for the JPM and XOM stocks together with their respective long-run MIDAS components obtained under the MMReDCC model. The forecast exercise is performed in three different ways. Firstly, we assess the predictive performance of the models under different loss functions and evaluate the significance of loss function differences by means of the Model Confidence Set (MCS) approach. Secondly, we evaluate the out-of-sample hedging performance of the different models by computing minimum variance and global minimum variance portfolios. Last, we test the models ability to accurately forecast portfolio Value-at-Risk.

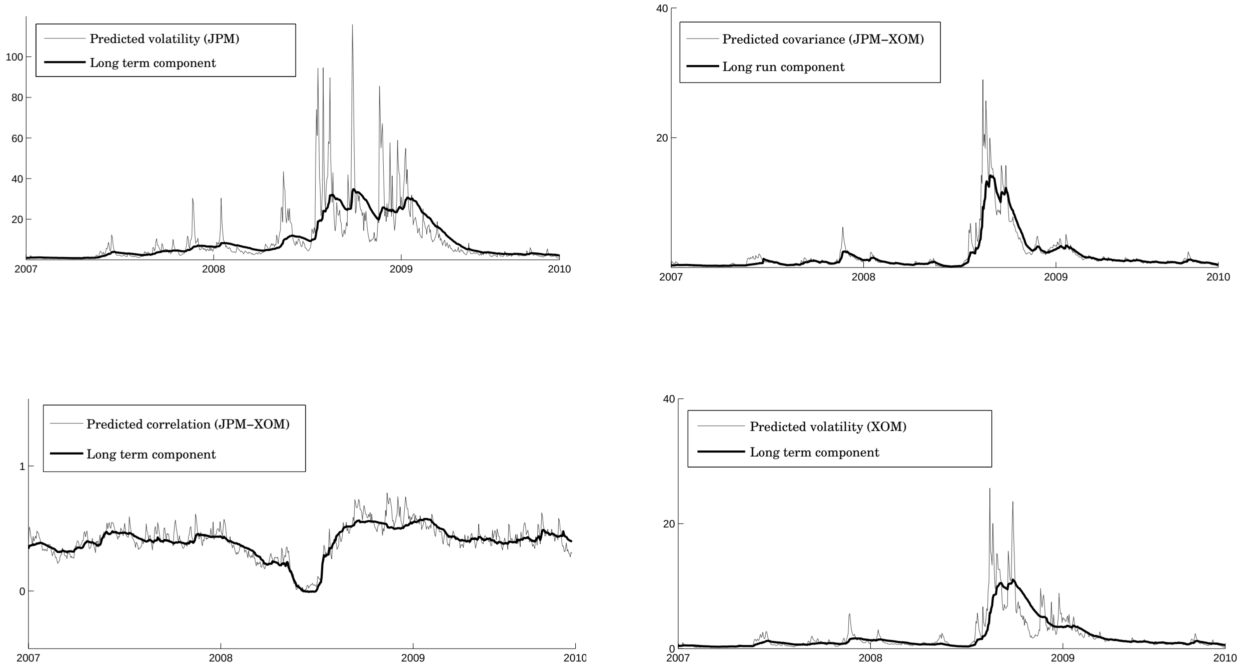

Figure 3: Predicted variance, covariance and correlation of JPM and XOM stocks from MMReDCC

4.3.1 Model Confidence Set

The Model Confidence Set (MCS) of Hansen et al. (2011) is the first method employed to compare the forecasting performance of the proposed models. Let M^0 denote the initial set of models for which we compute the series of one-step ahead conditional covariance forecasts for period t, denoted by H_t^i, where i denotes the i-th model. The MCS is based on an iterative procedure that requires sequential testing of equal predictive accuracy (EPA); this implies that the set of candidate models is sequentially trimmed by deleting those that are found to be statistically inferior within M^0. At a given level of confidence, the MCS contains the single model or the set of models having the best forecasting performance. The advantage of this procedure is that it does not necessarily require to select a privileged benchmark model.

At the heart of the method there is a forecast loss measure. The MCS final selection is based on the ordering implied by the loss function used to evaluate the deviations of each model predictions from the true conditional covariance matrix, denoted by Σ_t. Given that the true conditional covariance is latent even ex-post, we rely on an unbiased proxy of Σ_t. Our choice falls on the 5-minutes realized covariance estimator, $\hat{\Sigma}_t$, which is a more efficient estimator than the one based on the outer product of returns under the fairly general assumptions of absence of microstructure noise and other biases; see for example Barndorff-Nielsen & Shephard (2001), Aït-Sahalia et al. (2005), and Zhang (2011).

As regards the choice of loss functions, recent research on the consistent ranking of volatility forecasts by Patton (2011), Patton & Sheppard (2009), and Laurent et al. (2013) has highlighted that care needs to be taken during the selection process in order to avoid unintended results. We therefore employ matrix loss functions that are robust to noisy proxies. In other words, on average, they are expected to provide the same ranking between two forecasts independently of whether the true conditional covariance or a conditionally unbiased proxy is used. We opt for using several robust loss functions instead of a single one, in order to assess the sensitivity of the MCS to different functions. They correspond to Euclidean and Frobenius distances, Mean Square Forecast Error (MSFE), QLIKE, Stein and von Neumann divergence (VND). Their definition is reminded in the Web Appendix.

Table 4: Model confidence sets at 90% level.

Models	Euclidean	Frobenius	MSFE	QLIKE	Stein	VND	Performance
1 - AMReDCC		✓	✓				33
2 - AMReDECO		✓	✓				33
3 - AMAReDCC		✓	✓				33
4 - AMAReDECO		✓	✓				33
5 - AMReDCC-P		✓	✓			✓	50
6 - AMReDECO-P		✓	✓				33
7 - AMAReDCC-P		✓	✓				33
8 - AMAReDECO-P							0
9 - NPReDCC	✓	✓	✓			✓	67
10 - NPReDECO							0
11 - NPReBEKK							0
12 - MMReDCC				✓	✓	✓	50
13 - MMReDECO				✓	✓		33
14 - MMReBEKK	✓	✓	✓			✓	67
15 - MMAReDCC						✓	0
16 - MMAReDECO							0
17 - MMAReBEKK		✓	✓				33
18 - CReDCC		✓	✓				33
19 - CReDECO							0
20 - CReBEKK	✓	✓	✓				50
21 - RDCC							0

Notes – MCS *p*-values are provided in Table C3 of the Web Appendix. Loss functions defined in Section 4.3.1. 'Performance' is the percentage of inclusion of each model in the MCS across the six loss functions.

Table 5: Model confidence set at 75% level.

Models	Euclidean	Frobenius	MSFE	QLIKE	Stein	VND	Performance
1 - AMReDCC							0
2 - AMReDECO							0
3 - AMAReDCC							0
4 - AMAReDECO							0
5 - AMReDCC-P						✓	17
6 - AMReDECO-P							0
7 - AMAReDCC-P							0
8 - AMAReDECO-P							0
9 - NPReDCC	✓	✓	✓			✓	67
10 - NPReDECO							0
11 - NPReBEKK							0
12 - MMReDCC				✓	✓	✓	50
13 - MMReDECO				✓	✓		33
14 - MMReBEKK						✓	17
15 - MMAReDCC							0
16 - MMAReDECO							0
17 - MMAReBEKK							0
18 - CReDCC							0
19 - CReDECO							0
20 - CReBEKK							0
21 - RDCC							0

Notes – MCS *p*-values are provided in Table C4 of the Web Appendix. See also note of Table 4.

We follow Hansen et al. (2011) and compute the MCS at both the 75% and 90% confidence levels. The block-length bootstrap parameter and the number of bootstrap samples used to obtain the distribution under the null are set equal to 2 and 10000, respectively.

The MCS results at the 90% confidence level are reported in Table 4, and those at the 75% level in Table 5. In both tables, the last column shows a summary measure of model performance defined as the percentage of inclusion in the MCS across the six loss functions. At the 90% level, two benchmark models, CReDCC and CReBEKK, are included in the MCS resulting from the Frobenius and MSFE loss functions, and CReBEKK is included also in the Euclidean MCS. At the 75%, no benchmark model is included in any MCS. These result provide reasonable support to the conjecture that accounting for a time-varying long run component may contribute to improve forecasting performance.

At the 90% level, at least two models with time-varying long run components are included in the MCS for all loss functions. The highest number of models (ten) is included for the Frobenius and the MSFE loss functions. That number is considerably reduced for the other loss functions and this is also the case for all loss functions at the 75% confidence level. The most striking result is the inclusion of the NPReDCC model in the MCS of four loss functions at both levels, and at 75% it is the single model of the MCS based on the Euclidian, Frobenius, and MSFE functions. Multiplicative parametric models slightly outperform the additive models, except at the 90% level for the Frobenius and MSFE functions. DCC and BEKK-type models are more often selected than DECO-type models.

4.3.2 Out-of-sample hedging performance

In choosing among different competing models, practitioners are willing to employ the best one whose performance can be evaluated in an economically meaningful way. In portfolio management, for example, they are interested in the model providing portfolios with the lowest variance among a set of models. We accomplish this kind of comparison adopting the method proposed by Engle & Colacito (2006) pertaining to minimum variance portfolio management. Namely, by using out-of-sample covariance forecasts from the set of competing models, we form both global minimum variance (GMV) portfolios and minimum variance (MV) portfolios.

The GMV portfolio weights are obtained as solution to the optimization problem

$$\min_{w_t} w_t' H_t w_t \quad s.t. \sum_{j=1}^{n} w_{t,j} = 1,$$

where w_t is the vector of portfolio weights for time t chosen at time $t-1$, H_t denotes the conditional covariance forecast from a generic model, and the only requirement is on the vector of weights to sum up to unity. The MV portfolio is achieved by adding an additional constraint on the vector of expected returns μ, i.e. $w_t'\mu \geq q$. As pointed out by Engle & Colacito (2006), the portfolio volatility is smallest for the correctly specified covariance matrix for any vector of expected returns. Hence, for computational ease, we set the conditional mean return vector equal to the historical mean, and like Engle & Kelly (2012), we impose a target expected annual return of 10%.

As a result, we get series of 712 out-of-sample GMV and MV daily portfolio returns and their corresponding daily variances for each model. According to the initial statement, a superior model is required to produce optimal portfolios with lower variance realizations. In order to test for the significance of the differences between portfolio variances, we employ a Diebold and Mariano (2002) test between the overall best performing model among the 21 models and the first best model in the benchmark group. Table 6 reports standard deviations of the forecasted portfolio returns time series along with each model position in the ranking. Results for the GMV portfolios are shown in the second column of the table. Noticeably, there are no benchmark models up to the eighth position. The five best performing models belong to the multiplicative class, and the MMReDCC achieves the lowest variance GMV portfolio with a standard deviation of 1.0711. This improves at the 1% significance level over the first best benchmark, the CReDCC, which achieves a standard deviation of 1.0815. A plausible reason for this difference stems from the comparison of the correlations extracted from both models, as shown in Figure 4. Apparently, the CReDCC model tends to underestimate correlations especially during the higher volatile period, thus leading to a less appreciable gain from portfolio diversification. The other constant long term models have broadly the same

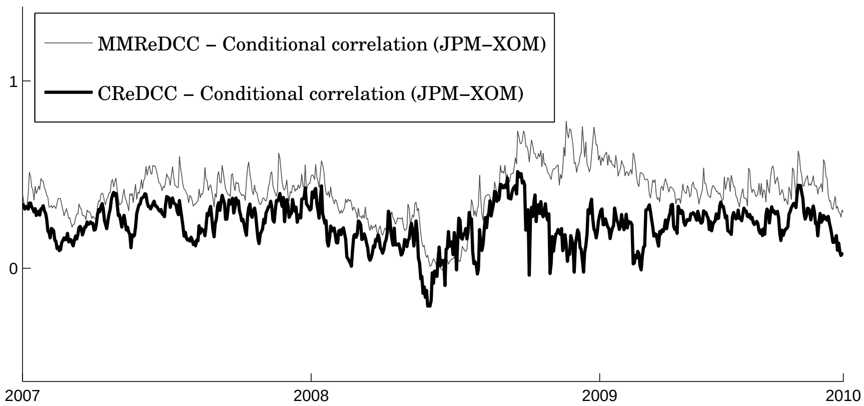

Figure 4: Comparison of predicted correlations of JPM-XOM stock from MMReDCC and CReDCC models

inferior performance, except that the CReBEKK model slightly improves over its competitors involving a BEKK-type structure. Not really surprisingly, as it reminds the finding obtained in the MCS evaluation, the RDCC closes the ranking with the highest standard deviation of 1.1218. Similar results are found for the MV portfolios in the fourth column. The MMReDCC model confirms its predominance (1.0958) and significantly improves over the first best constant long run model (CReBEKK, 1.1102) at the 1% significance level. The other benchmarks are all ranked in the last positions.

Briefly, we can interpret minimum variance portfolio results from this forecasting exercise as providing clear evidence that there can be hedging benefits from employing models that account for a time-varying long run component. In some cases, these benefits can be remarkable, depending on the chosen long run component-type structure or the short run multivariate specification.

4.3.3 Portfolio VaR forecasting

In this last application, we consider the forecasting of portfolio Value-at-Risk (VaR). The aim is to study the possible efficiency gain of using the time-varying long run component models over benchmarks for one-step-ahead VaR predictions. Our analysis is concerned with the forecasting of the *long* side of the daily VaR. This corresponds to the VaR level for traders having long positions in their asset holdings, and thus the predictive power of a specific model is related to its ability to model large negative returns. For simplicity, we abstract from the Markovitz

Table 6: Out-of-sample hedging performance

Models	GMV		MV	
	std.	**rank**	**std.**	**rank**
1 - AMReDCC	1.07963	(7)	1.10497	(8)
2 - AMReDECO	1.08335	(9)	1.11160	(14)
3 - AMAReDCC	1.08545	(13)	1.11072	(13)
4 - AMAReDECO	1.08672	(15)	1.11683	(15)
5 - AMReDCC-P	1.07792	(6)	1.10341	(6)
6 - AMReDECO-P	1.08590	(14)	1.11440	(16)
7 - AMAReDCC-P	1.08487	(12)	1.10968	(10)
8 - AMAReDECO-P	1.08782	(16)	1.11441	(17)
9 - NPReDCC	1.07190	(2)	1.09910	(4)
10 - NPReDECO	1.08404	(11)	1.10465	(7)
11 - NPReBEKK	1.11439	(20)	1.13175	(20)
12 - MMReDCC	**1.0711** **	(1)	**1.0958** **	(1)
13 - MMReDECO	1.07307	(3)	1.09874	(3)
14 - MMReBEKK	1.08886	(18)	1.10897	(9)
15 - MMAReDCC	1.07308	(4)	1.09687	(2)
16 - MMAReDECO	1.07790	(5)	1.10264	(5)
17 - MMAReBEKK	1.08958	(19)	1.11053	(12)
18 - CReDCC	1.08149	(8)	1.11168	(18)
19 - CReDECO	1.08357	(10)	1.13073	(19)
20 - CReBEKK	1.08858	(17)	1.11026	(11)
21 - RDCC	1.12175	(21)	1.14693	(21)

Notes – The table reports standard deviations of the portfolios return time series. The best performing model in each column is in bold. A Diebold-Mariano test is performed to compare the variances achieved by the best model against the first best performing model within the benchmarks in the same column. The best model is accompanied by * or ** if the difference is significant at the 5% or 1% level, respectively.

optimization setting employed in the previous application, considering only equally-weighted portfolios.

For each model, the portfolio VaR at level α on day t, conditional on the information available at time $t-1$, is computed as:

$$VaR_t(\alpha) = z_\alpha \sqrt{w' H_t w},$$

where w is the given n-dimensional vector of equal weights, H_t is the forecasted conditional covariance matrix for a generic model, and z_α is the $\alpha\%$ left-quantile of the standard normal distribution. The same analysis was done assuming the more flexible Student distribution, but as this did not lead to significant improvements, results are not reported. VaR at levels α equal to $5\%, 2.5\%$ and 1% are forecasted, and their performance is then assessed using two statistical backtesting methods. First, the Likelihood Ratio Conditional Coverage (LR_{cc}) test of Christoffersen (1998) is used; its construction relies on the so-called hit function, or indicator function, obtained as follows:

$$I_t(\alpha) = \begin{cases} 1 \text{ if } w'r_t \leq VaR_t(\alpha) \\ 0 \text{ if } w'r_t > VaR_t(\alpha). \end{cases}$$

According to Finger (2005), good VaR models are capable of reacting to changing volatility and correlations in a way that exceptions occur independently of each other, whereas bad models tend to produce a sequence of consecutive exceptions. Christoffersen's test accounts for both properties of a good VaR model, namely the correct failure rate and the independence of exceptions. The LR_{cc} test statistic is χ^2_1 distributed. The second method is the regression-based test of Engle & Manganelli (2004), also known as the Dynamic Quantile (DQ) test. Instead of directly considering the hit sequence, the test is based on its associated quantile process $H_t(\alpha) = I_t(\alpha) - \alpha$, formally expressed as

$$H_t(\alpha) = \begin{cases} 1 - \alpha \text{ if } I_t = 1 \\ -\alpha \text{ if } I_t = 0. \end{cases}$$

Its purpose it to link the current margin exceedances to past violations or past information and subsequently testing for different restrictions on the parameters of the regression. We run the regression $H_t(\alpha) = \delta + \sum_{k=1}^{K} \beta_k H_{t-k}(\alpha) + \epsilon_t$ for $K = 3$ and we test the joint hypothesis $H_0(DQ_{cc}) : \delta = \beta_1 = ... = \beta_K = 0$. This assumption coincides with the null of Christoffersen's LR_{cc} test. It is also possible to split the test and separately test the independence hypothesis and the unconditional coverage hypothesis, respectively as $H_0(DQ_{ind}) : \beta_1 = ... = \beta_K = 0$ and $H_0(DQ_{uc}) : \delta = 0$. Empirical results from the tests are given in Table 7. For each VaR level, we report test statistics along with the corresponding p-values. The first column of each panel depicts the results for the LR_{cc} test while the last three columns show the results for the DQ_{uc}, DQ_{ind} and DQ_{cc} tests. Across the different VaR panels, the LR_{cc} and DQ_{cc} tests basically tell the same story. Rejections of the first test at the 5% level correspond to rejections of the second, with very few exceptions.

Of main interest, the last four rows of the table report test statistics and p-values from the benchmark models. Results are quite homogeneous among the three VaR panels and, at least for the multiplicative models, they are considerably inferior to those obtained by the corresponding time-varying counterparts.

Irrespective of their model structure, the tests for the benchmark models lead to rejections in a vast majority of cases. On the contrary, the tests for the multiplicative component models show a remarkably better performance. Both the nonparametric and the parametric versions pass all the tests for $\alpha = 5\%$ showing occasional rejections of the DQ_{uc} test for the most extreme quantiles. Apparently, the flexibility of the multiplicative structure allows to adequately model the left tail of the distribution and to deliver superior VaR forecasts. This holds for any type of short term (DCC, BEKK DECO) specification.

It clearly appears that the additive class fails in predicting the VaR adequately. This is particularly visible for the 5% VaR, where the p-values for the null hypothesis of the various tests are often smaller than 0.05 for most models.

However, by looking at the results of the DQ_{ind} test across the board, violations do not appear to be dependent. For VaR at levels $\alpha = 2.5\%$ and $\alpha = 1\%$, models including a DCC specification show a slight improvement in capturing the conditional coverage of both tests, but the additive class still appears to be almost uniformly rejected. The only exception is the AMAReDCC model, which passes all the tests with a single rejection at the 5% level for the 1% VaR.

The conclusion that can be drawn is that, for equally weighted portfolios, reliable VaR forecasts can be obtained under the simple assumption of conditionally normally standardized portfolio returns, by using time-varying long run component models that include an appropriately chosen long term structure.

5 Conclusions

We propose a new set of component models allowing for time variation in the long run levels of realized variances and correlations. Our modeling framework allows for the secular component to enter the model structure either in additive fashion, as a time-varying intercept in the conditional correlation model, or as a multiplicative factor. In the latter case, it is specified parametrically, using a MIDAS specification, or non-parametrically, by means of a matrix-variate smoother.

As a general finding, additive-type models have good in-sample fits while multiplicative models tend to be preferred out-of-sample. In all cases, the choice of the multivariate GARCH specification plays a crucial role for accurately modeling the short-term dynamics and, among the three possibilities proposed, the DCC appears to be prevailing.

In the empirical application we illustrate the potential benefits of using a time-varying long run component instead of a constant one. When the dimension of the application is moderately small, such as ten assets, estimation can be performed by maximizing a Wishart quasi likelihood function in one step. This yields computationally tractable, consistent and asymptotically

Table 7: Likelihood Ratio Test and Dynamic Quantile Test Results

Models	VaR 1%				VaR 2.5%				VaR 5%			
	LR	DQ			LR	DQ			LR	DQ		
	cc	uc	ind	cc	cc	uc	ind	cc	cc	uc	ind	cc
1 - AMReDCC	3.94 (0.14)	5.09 **(0.02)**	0.44 (0.50)	5.30 (0.07)	2.65 (0.27)	3.22 (0.07)	1.31 (0.25)	4.32 (0.11)	8.18 **(0.02)**	8.55 **(0.00)**	1.42 (0.23)	9.43 **(0.01)**
2 - AMReDECO	8.26 **(0.02)**	11.60 **(0.00)**	0.84 (0.36)	12.10 **(0.00)**	2.65 (0.26)	3.22 (0.07)	1.31 (0.25)	4.32 (0.11)	10.20 **(0.01)**	8.90 **(0.00)**	3.40 (0.06)	11.44 **(0.00)**
3 - AMAReDCC	2.80 (0.24)	3.50 (0.06)	0.35 (0.55)	3.74 (0.15)	2.00 (0.36)	2.38 (0.12)	1.16 (0.28)	3.39 (0.18)	5.51 (0.07)	5.65 **(0.02)**	0.87 (0.35)	6.23 (0.05)
4 - AMAReDECO	6.68 **(0.04)**	9.17 **(0.00)**	0.68 (0.41)	9.35 **(0.01)**	4.22 (0.12)	5.28 **(0.02)**	1.66 (0.20)	6.57 **(0.04)**	8.18 **(0.02)**	8.55 **(0.00)**	1.42 (0.23)	9.43 **(0.01)**
5 - AMReDCC-P	1.83 (0.40)	2.21 (0.14)	0.27 (0.60)	2.41 (0.30)	2.65 (0.26)	3.22 (0.07)	1.31 (0.25)	4.32 (0.11)	7.24 **(0.03)**	7.52 **(0.01)**	1.22 (0.27)	8.29 **(0.02)**
6 - AMReDECO-P	6.68 **(0.04)**	9.17 **(0.00)**	0.68 (0.41)	9.53 **(0.01)**	2.65 (0.27)	3.22 (0.07)	1.31 (0.25)	4.32 (0.11)	12.40 **(0.00)**	11.20 **(0.00)**	0.13 (0.71)	14.13 **(0.00)**
7 - AMAReDCC-P	3.94 (0.14)	5.09 **(0.02)**	0.44 (0.51)	5.37 (0.07)	2.01 (0.37)	2.39 (0.12)	1.16 (0.28)	3.39 (0.18)	5.52 (0.06)	5.65 **(0.02)**	0.87 (0.34)	6.23 **(0.04)**
8 - AMAReDECO-P	6.68 **(0.04)**	9.17 **(0.00)**	0.68 (0.41)	9.53 **(0.01)**	3.40 (0.18)	4.18 **(0.04)**	1.48 (0.22)	5.38 (0.07)	9.07 **(0.01)**	10.42 **(0.00)**	0.51 (0.47)	10.59 **(0.01)**
9 - NPReDCC	1.83 (0.40)	2,21 (0.13)	0.27 (0.60)	2.41 (0.29)	4.22 (0.12)	5.28 **(0.02)**	1.66 (0.19)	6.57 **(0.04)**	3.51 (0.17)	3.88 (0.05)	0.01 (0.94)	3.88 (0.14)
10 - NPReDECO	3.94 (0.13)	5.09 **(0.02)**	0.44 (0.50)	5.37 (0.07)	1.42 (0.49)	1.67 (0.19)	1.02 (0.31)	2.59 (0.27)	2.73 (0.25)	2.74 (0.09)	0.36 (0.54)	3.00 (0.22)
11 - NPReBEKK	3.84 (0.14)	1.83 (0.17)	6.42 **(0.01)**	8.57 (0.01)	4.22 (0.12)	4.89 **(0.03)**	0.00 (0.97)	4.91 (0.09)	4.18 (0.12)	2.87 (0.09)	1.70 (0.19)	4.35 (0.11)
12 - MMReDCC	1.83 (0.40)	2.21 (0.14)	0.27 (0.60)	2.41 (0.29)	0.55 (0.76)	0.63 (0.42)	0.77 (0.37)	1.37 (0.50)	5.65 (0.06)	3.74 (0.05)	2.12 (0.14)	6.01 (0.05)
13 - MMReDECO	0.10 (0.95)	0.11 (0.73)	0.10 (0.74)	0.21 (0.89)	0.94 (0.62)	1.09 (0.29)	0.89 (0.34)	1.92 (0.38)	4.18 (0.12)	2.87 (0.09)	1.70 (0.19)	4.35 (0.11)
14 - MMReBEKK	1.04 (0.59)	1.21 (0.27)	0.20 (0.65)	1.38 (0.50)	2.65 (0.27)	3.22 (0.07)	1.31 (0.25)	4.32 (0.11)	2.73 (0.25)	2.74 (0.09)	0.35 (0.55)	3.00 (0.22)
15 - MMAReDCC	2.80 (0.24)	3.50 (0.06)	0.35 (0.55)	3.74 (0.15)	0.56 (0.76)	0.63 (0.43)	0.77 (0.38)	1.37 (0.50)	4.19 (0.12)	2.87 (0.09)	1.70 (0.19)	4.35 (0.11)
16 - MMAReDECO	1.04 (0.59)	1.21 (0.27)	0.20 (0.65)	1.38 (0.50)	0.94 (0.62)	1.09 (0.29)	0.89 (0.35)	1.92 (0.38)	4.18 (0.12)	2.87 (0.09)	1.70 (0.19)	4.35 (0.11)
17 - MMAReBEKK	1.83 (0.40)	2.21 (0.13)	0.27 (0.60)	2.41 (0.29)	2.65 (0.27)	3.22 (0.07)	1.31 (0.25)	4.33 (0.11)	1.51 (0.46)	0.62 (0.43)	0.87 (0.35)	1.45 (0.48)
18 - CReDCC	9.97 **(0.01)**	14.46 **(0.00)**	1.00 (0.31)	14.90 **(0.00)**	13.36 **(0.00)**	18.70 **(0.00)**	3.65 (0.05)	20.77 **(0.00)**	14.62 **(0.00)**	17.19 **(0.00)**	1.39 (0.23)	17.68 **(0.00)**
19 - CReDECO	24.90 **(0.00)**	42.67 **(0.00)**	2.85 (0.09)	43.40 **(0.00)**	30.15 **(0.00)**	47.56 **(0.00)**	7.89 **(0.01)**	50.66 **(0.00)**	35.03 **(0.00)**	45.97 **(0.00)**	2.31 (0.12)	46.09 **(0.00)**
20 - CReBEKK	1.83 (0.40)	2.21 (0.13)	0.27 (0.60)	8.81 **(0.03)**	4.22 (0.12)	5.28 **(0.02)**	1.66 (0.19)	6.57 **(0.03)**	5.51 (0.06)	5.65 **(0.01)**	0.87 (0.34)	6.23 **(0.04)**
21 - RDCC	5.23 (0.07)	6.98 **(0.01)**	0.55 (0.45)	7.30 **(0.02)**	9.43 **(0.01)**	12.68 **(0.00)**	2.78 (0.09)	14.46 **(0.00)**	18.52 **(0.00)**	22.12 **(0.00)**	2.15 (0.14)	22.80 **(0.00)**

Notes – The table reports test statistics of the Likelihood Ratio (LR) Conditional Coverage Test and of the Dynamic Quantile (Unconditional Coverage, Independence and Conditional Coverage) test for the equal weighted portfolios Value-at-Risk at confidence levels α=1%, 2.5% and 5%. Corresponding p-values in brackets; p-values in bold denote significance at the 5% level.

efficient estimates of the parameters. Estimation results show that time varying component models deliver better full-sample likelihood fits as well as lower AIC and BIC values.
Their forecasting ability is then compared by means of three applications. Firstly, the MCS approach is used to identify the best performing models given a set of consistent loss functions. Results are in favour of the time-varying long run models, especially at the 75% MCS, as none of the benchmarks is included. Secondly, we construct (global) minimum variance portfolios to assess the models out-of-sample hedging performance. Again, constant long run models are often outperformed, suggesting that there can be hedging benefits from employing models which account for a time-varying long run component. Last, we forecast $1\%, 2.5\%$ and 5% Value-at-Risk assuming equally weighted portfolios under the assumption of standard normal quantiles. The benchmark models appear to deliver VaR forecasts that are considerably inferior to those obtained by their corresponding more flexible counterparts.

Overall, our empirical results provide some evidence in favour of the hypothesis that accounting for time variation in levels of volatilities and correlations contributes both to improve in-sample fit and to provide more accurate out-of-sample one-step ahead forecasts.

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Web Appendix to

Forecasting Comparison of Long Term Component Dynamic Models For Realized Covariance Matrices

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Appendix A Descriptive statistics

Table A1: Descriptive statistics of realized variances.

Stock	Mean	Max.	Min.	Std.dev.	Skewness	Kurtosis
Estimation sample: February 1, 2001 to December 31, 2006 (1528 observations)						
BAC	1.31	19.35	0.07	1.88	4.66	32.54
JPM	2.88	168.72	0.11	6.45	14.86	329.89
IBM	1.60	40.69	0.08	2.27	6.23	73.62
MSFT	2.19	29.65	0.10	2.86	3.98	26.93
XOM	1.59	40.89	0.13	1.98	8.29	126.45
AA	2.85	30.57	0.29	2.71	3.71	24.24
AXP	2.33	62.94	0.08	4.01	6.23	65.13
DD	1.83	28.93	0.16	2.07	4.98	46.68
GE	1.98	55.74	0.10	3.06	6.49	80.14
KO	1.29	22.21	0.04	1.60	5.57	52.96
Forecasting sample: January 1, 2007 to December 31, 2009 (712 observations)						
BAC	14.36	277.31	0.10	27.69	4.07	25.22
JPM	9.74	176.48	0.16	16.32	4.74	34.81
IBM	2.65	57.54	0.10	4.88	5.86	49.89
MSFT	3.02	43.11	0.08	4.32	4.65	31.49
XOM	3.12	115.38	0.17	6.66	9.28	129.77
AA	9.44	160.24	0.49	14.36	4.82	36.89
AXP	8.91	201.88	0.24	14.14	6.09	64.94
DD	4.02	63.87	0.17	5.60	4.75	35.36
GE	5.81	114.26	0.18	11.37	4.66	31.05
KO	1.69	56.51	0.12	3.23	8.79	125.68
Full sample: February 1, 2001 to December 31, 2009 (2240 observations)						
BAC	5.46	277.31	0.07	16.82	7.17	72.51
JPM	5.06	176.48	0.11	11.10	7.53	84.62
IBM	1.93	57.54	0.08	3.36	7.32	84.96
MSFT	2.45	43.11	0.08	3.41	4.73	36.29
XOM	2.07	115.38	0.13	4.16	13.29	287.87
AA	4.95	160.24	0.29	8.94	7.63	92.05
AXP	4.42	201.88	0.08	9.16	8.53	133.60
DD	2.53	63.87	0.16	3.73	6.44	68.47
GE	3.20	114.26	0.10	7.12	7.23	75.42
KO	1.41	56.51	0.04	2.26	9.71	180.08

Notes – Descriptive statistics of the realized variances of the stocks used in the empirical application. The three panels report the statistics for the in-sample period, the out-of-sample period and the full sample period, respectively.

Appendix B Full sample estimates

Table B2: QML parameter estimates

Models	Kernel	MIDAS estimates		GARCH estimates			Correlation estimates		
	h	θ	ω_{2s}	$\bar{m}$	γ	δ	α	β	ω_{2r}
1 - AMReDCC		0.68 (0.080)	7.51 (2.506)	0.34 (0.09)	0.33 (0.03)	0.50 (0.06)	0.12 (0.00)	0.56 (0.15)	12.05 (2.84)
2 - AMReDECO		0.64 (0.30)	7.19 (3.20)	0.40 (0.46)	0.35 (0.05)	0.50 (0.09)	0.24 (0.04)	0.63 (0.18)	10.73 (14.47)
3 - AMAReDCC		0.02 (0.00)	8.34 (8.55)	0.53 (0.35)	0.33 (0.04)	0.56 (0.08)	0.27 (0.06)	0.70 (0.08)	12.10 (3.83)
4 - AMAReDECO		0.02 (0.01)	4.91 (5.11)	0.67 (0.50)	0.37 (0.04)	0.55 (0.05)	0.47 (0.09)	0.51 (0.10)	20.6 (9.93)
5 - AMReDCC-P		0.61 (0.11)	6.68 (2.50)	0.42 (0.13)	0.33 (0.03)	0.52 (0.06)	0.09 (0.00)	0.89 (0.00)	2.91 (1.44)
6 - AMReDECO-P		0.60 (0.27)	7.84 (16.40)	0.52 (0.28)	0.35 (0.053)	0.44 (0.12)	0.25 (0.05)	0.70 (0.07)	2.46 (11.2)
7 - AMAReDCC-P		0.03 (0.01)	7.84 (23.78)	0.72 (1.89)	0.35 (0.12)	0.58 (0.09)	0.25 (0.11)	0.73 (0.13)	11.75 (24.29)
8 - AMAReDECO-P		0.02 (0.00)	5.42 (7.30)	0.74 (0.19)	0.37 (0.03)	0.55 (0.06)	0.40 (0.19)	0.60 (0.21)	27.30 (9.29)
9 - NPReDCC	0.078				0.37 (0.03)	0.60 (0.04)	0.08 (0.00)	0.88 (0.00)	
10 - NPReDECO	0.078				0.36 (0.03)	0.60 (0.04)	0.20 (0.02)	0.74 (0.03)	
11 - NPReBEKK	0.078						0.23 (0.01)	0.74 (0.01)	
12 - MMReDCC		0.97 (0.42)			0.23 (0.04)	0.50 (0.17)	0.07 (0.01)	0.82 (0.14)	7.21 (2.80)
13 - MMReDECO		0.97 (0.07)			0.39 (0.09)	0.47 (0.13)	0.19 (0.08)	0.72 (0.13)	13.87 (1.72)
14 - MMReBEKK		0.87 (0.04)					0.20 (0.02)	0.55 (0.07)	19.89 (7.63)
15 - MMAReDCC		0.04 (0.00)			0.38 (0.04)	0.52 (0.05)	0.06 (0.00)	0.87 (0.01)	5.79 (0.62)
16 - MMAReDECO		0.04 (0.00)			0.37 (0.05)	0.48 (0.13)	0.20 (0.08)	0.72 (0.19)	12.17 (5.48)
17 - MMAReBEKK		0.04 (0.00)					0.24 (0.01)	0.62 (0.02)	8.62 (1.01)
18 - CReDCC					0.22 (0.02)	0.70 (0.02)	0.27 (0.00)	0.65 (0.01)	
19 - CReDECO					0.14 (0.05)	0.79 (0.10)	0.32 (0.09)	0.70 (0.19)	
20 - CReBEKK							0.28 (0.03)	0.74 (0.03)	
21 - RDCC					0.32 (0.12)	0.56 (0.21)	0.19 (0.08)	0.70 (0.09)	

Notes – QML parameter estimates and corresponding robust standard errors in brackets. For the univariate MIDAS and GARCH estimates we report average values among series and corresponding mean asymptotic square errors (MASE). Asymptotic standard errors are calculated using the "sandwich" estimator of Bollerslev et al.(1988). The results are based on the full-sample dataset, i.e. from February 1, 2001 to December 31, 2009.

Appendix C Model Confidence Set

Appendix C.1 Loss functions

The set of robust loss functions used in the empirical application involves Euclidean and Frobenius distances, Mean Square Forecast Error (MSFE), QLIKE, Stein and von Neumann divergence (VND). Their definition is provided in the following table.

Loss Function	Formula	Type
Euclidean	$\text{vech}\left(\hat{\Sigma}_t - H_t\right)' \text{vech}\left(\hat{\Sigma}_t - H_t\right)$	Symmetric
Frobenius	$\text{tr}\left[\left(\hat{\Sigma}_t - H_t\right)' \left(\hat{\Sigma}_t - H_t\right)\right]$	Symmetric
MSFE	$\frac{1}{T}\text{vec}\left(\hat{\Sigma}_t - H_t\right)' \text{vec}\left(\hat{\Sigma}_t - H_t\right)$	Symmetric
QLIKE	$\log\lvert H_t\rvert + \text{vec}\left(H_t^{-1}\hat{\Sigma}_t\right)' \iota$	Symmetric
Stein	$\text{tr}(H_t^{-1}\hat{\Sigma}_t) - \log\lvert H_t^{-1}\hat{\Sigma}_t\rvert - n$	Asymmetric
VND	$\text{tr}\left(\hat{\Sigma}_t \log\hat{\Sigma}_t - \hat{\Sigma}_t \log H_t - \hat{\Sigma}_t + H_t\right)$	Asymmetric

Notes – ι denotes a vector of ones, T is the out-of-sample length and n is the number of assets.

In the table, $\hat{\Sigma}_t$ is the proxy used for the true conditional covariance matrix while H_t denotes each model predicted covariance matrix for the day t (the subscript i on H_t has been removed for simplicity).

The first three loss functions are symmetric quadratic loss functions based on the forecast error. The Euclidean distance only accounts for the unique elements of the covariance matrix while the Frobenius distance double counts the loss associated to the conditional covariances.

The last two loss functions belong to the family of Bregman matrix divergences, which generalize the squared Euclidean distance to a class of distances that all share similar properties. The Stein loss function is scale invariant, being based on the standardized forecast error (in matrix sense) and can be obtained as the objective function of the Wishart distribution, while the von Neumann divergence is a matrix generalization of Kullback-Leibler divergence. Both Stein and VND are asymmetric with respect to over/under predictions.

Appendix C.2 MCS p-values

Table C3: 90% MCS p-values

Models	Euclidean	Frobenius	MSFE	QLIKE	Stein	VND
1 - AMReDCC	0.0973	0.1799	0.1637	0.0067	0.0053	0.0987
2 - AMReDECO	0.0973	0.1799	0.1637	0.0000	0.0000	0.0000
3 - AMAReDCC	0.0973	0.1799	0.1637	0.0067	0.0001	0.0929
4 - AMAReDECO	0.0973	0.1799	0.1637	0.0000	0.0000	0.0000
5 - AMReDCC-P	0.0996	0.1927	0.1936	0.0067	0.0053	0.203
6 - AMReDECO-P	0.0608	0.1257	0.1214	0.0000	0.0000	0.0000
7 - AMAReDCC-P	0.0973	0.1799	0.1637	0.0003	0.0000	0.0929
8 - AMAReDECO-P	0.0608	0.0943	0.0854	0.0000	0.0000	0.0000
9 - NPReDCC	1.000	1.000	1.000	0.0000	0.0000	1.000
10 - NPReDECO	0.0608	0.0877	0.0854	0.0000	0.0000	0.0013
11 - NPReBEKK	0.0608	0.0943	0.0854	0.0000	0.0000	0.0013
12 - MMReDCC	0.0608	0.0943	0.0854	1.000	1.000	0.203
13 - MMReDECO	0.0608	0.0943	0.0854	0.2993	0.3005	0.0987
14 - MMReBEKK	0.1885	0.1927	0.1936	0.0000	0.0000	0.203
15 - MMAReDCC	0.0608	0.0943	0.0854	0.0067	0.0053	0.1612
16 - MMAReDECO	0.0608	0.0877	0.0854	0.0003	0.0001	0.0168
17 - MMAReBEKK	0.0996	0.1927	0.1936	0.0000	0.0000	0.0388
18 - CReDCC	0.0608	0.1257	0.1214	0.0000	0.0000	0.0168
19 - CReDECO	0.0608	0.0943	0.0854	0.0000	0.0000	0.0000
20 - CReBEKK	0.1885	0.206	0.1993	0.0000	0.0000	0.0987
21 - RDCC	0.0533	0.076	0.0697	0.0000	0.0000	0.0000

Notes – Entries are the p-values of the models included in the 90% MCS.

Table C4: 75% MCS p-values

Models	Euclidean	Frobenius	MSFE	QLIKE	Stein	VND
1 - AMReDCC	0.103	0.192	0.167	0.006	0.006	0.104
2 - AMReDECO	0.103	0.192	0.167	0.000	0.000	0.000
3 - AMAReDCC	0.103	0.192	0.167	0.006	0.006	0.094
4 - AMAReDECO	0.103	0.192	0.167	0.000	0.000	0.000
5 - AMReDCC-P	0.103	0.203	0.188	0.006	0.006	0.273
6 - AMReDECO-P	0.066	0.128	0.130	0.000	0.000	0.000
7 - AMAReDCC-P	0.103	0.192	0.167	0.001	0.001	0.094
8 - AMAReDECO-P	0.066	0.103	0.087	0.000	0.000	0.000
9 - NP-ReDCC	1.000	1.000	1.000	0.000	0.000	1.000
10 - NP-ReDECO	0.066	0.086	0.087	0.000	0.000	0.001
11 - NP-ReBEKK	0.066	0.103	0.087	0.000	0.000	0.001
12 - MMReDCC	0.066	0.103	0.087	1.000	1.000	0.273
13 - MMReDECO	0.066	0.103	0.087	0.309	0.289	0.104
14 - MMReBEKK	0.185	0.203	0.188	0.000	0.000	0.273
15 - MMAReDCC	0.066	0.103	0.087	0.006	0.006	0.181
16 - MMAReDECO	0.066	0.086	0.087	0.001	0.001	0.018
17 - MMAReBEKK	0.103	0.203	0.188	0.000	0.000	0.043
18 - CReDCC	0.066	0.128	0.130	0.000	0.000	0.018
19 - CReDECO	0.066	0.103	0.087	0.000	0.000	0.000
20 - CReBEKK	0.185	0.203	0.210	0.000	0.000	0.104
21 - RDCC	0.046	0.073	0.069	0.000	0.000	0.000

Notes – Entries are the p-values of the models included in the 75% MCS.

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